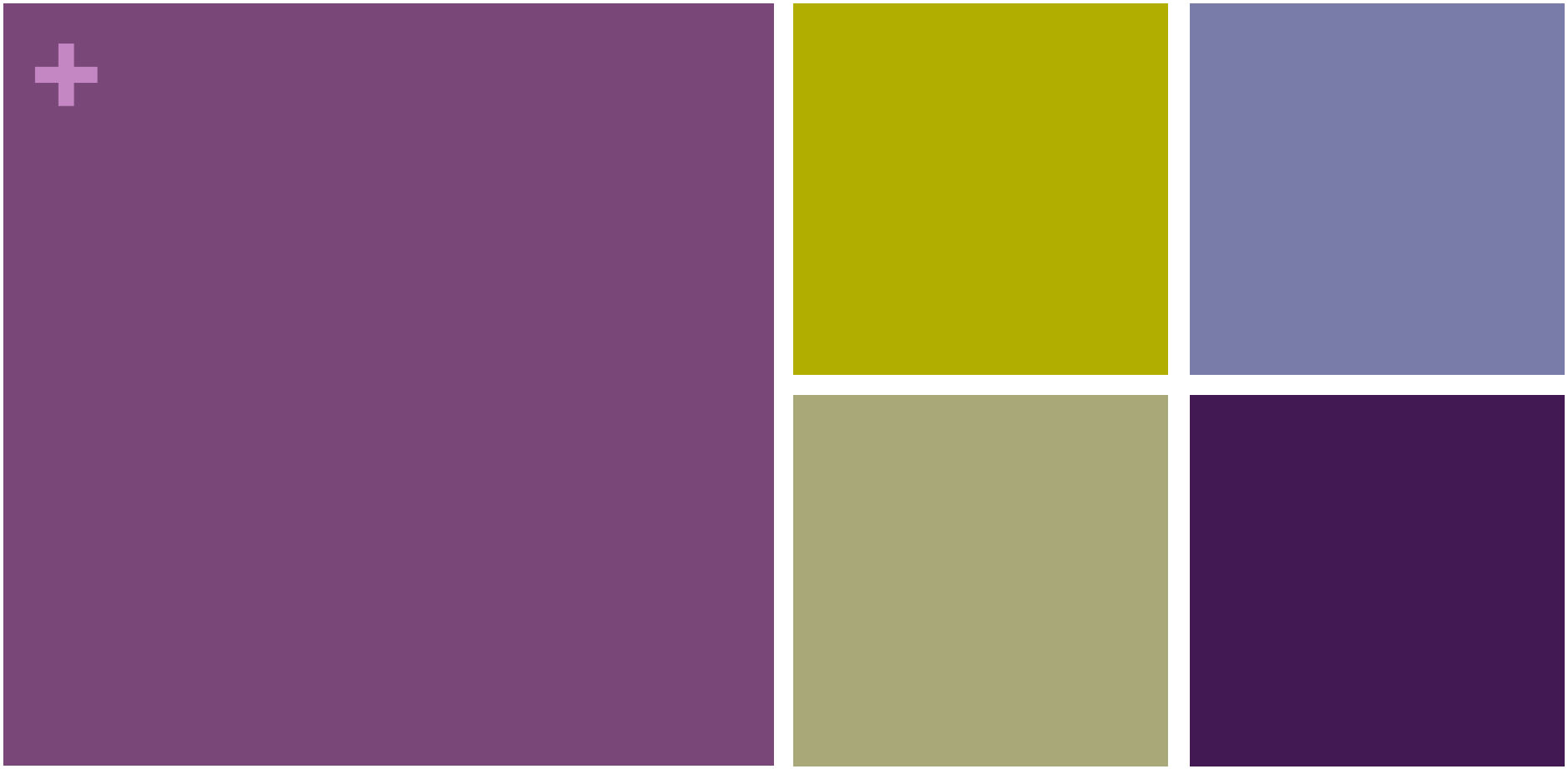

Propositional Logic Revision Tutorial

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Today's Objectives

- Propositions
- Complex Propositions
- Valid Propositions
- Correct or Incorrect?
- Is it a predicate?
- Assertions using Predicates

This tutorial assumes that you know about truth tables.

NB: different texts may use different symbols.

I am not an expert on this topic: this tutorial is for revision.

Is it a Proposition?

- Propositions:
  - Language independant.
  - Formed of statements.
  - Are either true or false as a fact
    - Not questions.
    - Not a test.
  - Simple statements are indivisible.
  - All statements made up of combinations of simple statements combined using logical operators.
- Propositional logic does not usually study the subject or predicates used within statements.

Question 1 – Is it a Proposition?

- For each of the following sentences, say whether they are propositions or not:
  - Should we go now?
  - My mum is taller than me
  - Everybody is happy
  - Are you happy?
  - Go away!

Answer 1 – Is it a Proposition?

- For each of the following sentences, say whether they are propositions or not:
  - Should we go now? — No: It is a question, not a statement.
  - My mum is taller than me — Yes: A statement that is true/false.
  - Everybody is happy — Yes: A statement that is true/false.
  - Are you happy? — No: This is a question.
  - Go away! — No: This is a test.

Complex Propositions

- Simple propositions are indivisible.
- Complex propositions are made up of simple or recursive complex propositions.
- Propositions must be combined using, or may be modified, using logical operators:

OR	\/	Disjunction
AND	/\	Conjunction
NOT	~	Negation
IF … THEN	->	Implication
IF AND ONLY IF	<->	Material Equivalence

Complex Propositions: Implication

A	B	A -> B
T	T	T
T	F	F
F	T	T
F	F	T

- If A is true, then B is true.
- BUT: B can still be true if A is false.
  - To obtain equivalence you need IF AND ONLY IF (<->)
  - Truth table same as above, except F -> T is F.

Question 2 – Complex Propositions

- Let P and Q be the propositions:
- P: Your car is out of petrol. Q: You can't drive your car.
- Write the following propositions using P and Q and logical connectives.
  - (a) Your car is not out of petrol.
  - (b) You can't drive your car if it is out of petrol.
  - (c) Your car is not out of petrol if you can drive it.

Answer 2 – Complex Propositions

- P: Your car is out of petrol. Q: You can't drive your car.
  - (a) Your car is not out of petrol.
    - ~P
  - (b) You can't drive your car if it is out of petrol.
    - P -> Q
  - (c) Your car is not out of petrol if you can drive it.
    - ~Q -> ~P

Question 3 – Valid Propositions

- For each of the following expressions, indicate whether they are valid propositions or not. If not, say why they are not valid propositions.
  - P ∧ ~Q
  - [[Q ∨ R] [P ∧ Q]]

Answer 3 – Valid Propositions

- For each of the following expressions, indicate whether they are valid propositions or not. If not, say why they are not valid propositions.
  - P ∧ ~Q
    - P AND NOT Q
    - Valid (Hint: Well Formed – we don't care about meaning)
  - [[Q ∨ R] [P ∧ Q]]
    - (Q OR R)(P AND Q)
    - Invalid (the two sub propositions are not combined with an operator.)

Question 4 – Correct or Incorrect?

- Indicate which of the following statements are correct and which ones are incorrect.
  - If R is True and Q is True, then $R \wedge Q$ is True.
  - If R is True and Q is False, then $\sim[R \wedge Q]$ is False

Answer 4 – Correct or Incorrect?

If R is True and Q is True, then R ∧ Q is True.

Yes. AND is true if both inputs are true:

R	Q	R∧Q					
T	T	T					

If R is True and Q is False, then ~[R ∧ Q] is False.

No. If any input is false then AND is false. Inversion results in true – so this is inaccurate.

R	Q	R∧Q	~R∧Q				
T	F	F	T				

Is it a predicate?

- Predicate logic a.k.a. first-order logic.
- Predicate logic extends propositional logic by allowing quantification. Quantification is not literal numbers.
- The quantification comes from operators. But predicates needed for association of propositions.
- Example:
  - Ben is a man. Paul is a man.
- In propositional logic, these are unconnected. But valid in terms of structure.
- Predicate logic links them: Man(Ben), Man(Paul).
  - We can then do things like ‘for every Man’…

Question 5 – Is it a predicate?

- For each of the following sentences, say whether they are predicates or not,
  - (i) $x^2 = 4$
  - (ii) My friend John is taller than 2.1 meters
  - (iii) $2 - y = ¼$
  - (iv) I am 80 years old
  - (v) $x^4 = 16$
  - (vi) My friend John is taller than 2.1 meters

Answer 5 – Is it a predicate?

- (i) $x^2 = 4$
  - No. '4' is not true or false.
  - Could take whole thing as a statement, but it is not quantified.
- (ii) My friend John is taller than 2.1 meters
  - Yes. Could be IsFriend(John), Tall(John)
- (iii) 2 – y = ¼
  - No. '2 – y' is not true or false.
- (iv) I am 80 years old
  - Yes. Could Be OverEighty(Me)
- (v) $x^4 = 16$
  - No. '16' not true or false.

Assertions using Predicates

Type	Symbol	Example
For all	∀	$\forall x(Dog(x) \rightarrow ChewsBones(x))$ For all x: if x is a dog then x chews bones.
There exists	∃	$\exists x(Dog(x) \wedge IsPink(x))$ There exists an x which is a dog and is pink.

It is not your job to actually prove these, just to specify them.

A program could obviously be written to support this. Probably using 'for' loops and data sets.

Prolog and SWI-Prolog are examples.

This is known as **declarative programming**, you feed in data and the equation and out pops the answer.

Contrast with **procedural programming**!

Question 6 – Assertions using Predicates

- Working with all the character of the "Simpsons", express the assertions given below as a proposition of predicate logic using the following predicates.
  - Father (x,y) x is y's father, or equivalently y is x's child.
  - Mother (x,y) x is y's mother, or equivalently y is x'd child
  - Sister (x,y): x is y's sister
- **Marge is Lisa's mother but she is not Homer's mother.**
- **There is a character in the Simpsons that is Lisa's mother and Bart's mother.**
- **There is a kid whose father is Homer and whose sister is Lisa.**
- Marge is Lisa's mother and Bart's mother
- There is character in the Simpsons that is Lisa's mother and Bart's mother
- There is a child whose father is Homer and whose brother is Bart

Answer 6 – Assertions using Predicates

- Father (x,y) x is y's father, or equivalently y is x's child.
- Mother (x,y) x is y's mother, or equivalently y is x'd child
- Sister (x,y): x is y's sister

- Marge is Lisa's mother but she is not Homer's mother.

$$Mother(Mrge,Lisa) \wedge \neg Mother(Mrge,Homer)$$

- There is a character in the Simpsons that is Lisa's mother and Bart's mother. (Let's assume the Universe is the Simpsons…)

$$\exists x(Mother(x,Lisa) \wedge Mother(x,Bart)$$

- There is a kid whose father is Homer and whose sister is Lisa.

$$\exists x(Father(Homer,x) \wedge Sister(x,Lisa)$$

Reading Material

- Go over the slides for the relevant elements of the course.
- Try reading this as well, for a different explanation:
  - http://www.iep.utm.edu/p/prop-log.htm
  - http://www.cs.odu.edu/~toida/nerzic/content/logic/pred_logic/intr_to_pred_logic.html
  - http://en.wikipedia.org/wiki/First-order_logic
  - Remember that predicate logic is an extension of propositional logic. Propositional logic deals with structure. Predicate logic adds quantifiers and association.

Exam Advice

- Questions about procedure and admin > Cath Ewan quickly.
- Help with a particular question or topic:
  - Java Café, Email the course lecturer, Talk to friends, Research online, Library, etc.
- Revision:
  - Divide up your time wisely. Leave slack for the weather/socials.
  - Find a method you feel comfortable with.
  - Keep away from distractions.
  - It is unlikely last minute revision will work well. Aim to relax on the night before the exam and have a small glance at relevant notes before the exam.
- Don't panic if one or two topics are not going well.

Finally…

- These slides will appear on the website:
  - http://www.tonychung.net/
- I am happy to answer questions and provide help over email for the rest of your course – but unfortunately I am away for the next five weeks.
- Good luck with the revision.
- When your exams are over, chill!
- Think about applications for summer internships in 2010. Some companies require that you apply a year ahead!